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Application Note

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How Many Bits are Enough?

The Trade-off Between High Resolution and Low Power Using Oversampling Modes

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1.0 Introduction

This application note is targeted to try to help answer the question “How many bits are enough” as well as several others that fall out of this same question. The intent is to educate customers on the details to understand more about their application, to help extract meaningful information from the data sheet and learn some of the techniques to get the most out of the sensor for the application while realizing the limitations of the sensor.

Often the customer will be looking for the resolution limitations of the accelerometer to determine if the sensor is capable of the application. Resolution is defined as the smallest detectable change in acceleration. This information can be extracted out of the number of bits of the accelerometer. This is part of the information required but it is not the only piece of information. The system noise performance must also be taken into account. Without a device to do noise measurements and experimentally test the resolution how can one calculate the expected values and what are some of the important parameters to consider that will affect the results?

If an accelerometer is advertised as being 12-bit, what does that mean? The first thing one should ask is what range is the 12 bits divided into. Twelve bits over an 8g range is not the same as 12 bits over a 2g range. Over an 8g range, 12 bits will give you 256 counts/g and over a 2g range, 12 bits will give you 1024 counts per g, which is 4 times more sensitivity! It is important to first understand what the *sensitivity* is of the device by looking at the total dynamic range that the accelerometer is measuring.

1.1 Key Words

Accelerometer, Static Acceleration, Tilt Angles, Portrait/Landscape Orientation, Embedded Algorithm MMA8450Q, Z-Angle Lockout, XYZ Output Data, Low Current Consumption, Motion and Tap Detection, Design Flexibility, Hysteresis, 3-axis Accelerometer, Offset Considerations, Sample Rate, Debounce

1.2 Summary

A. The effective number of bit calculations provides better insight into the resolution of the digitized signal.

B. Noise affects the resolution of the device. Calculations for the Root Mean Square (RMS) noise and Power Spectral Density (PSD) are valuable to understand the effective number of bits.

C. Oversampling can be used to improve the resolution of the device. The MMA8451, 2, 3Q provides 4 different oversampling schemes which trade-off between resolution and current consumption at varying sample rates.

2.0 MMA8451, 2, 3Q Consumer 3-axis Accelerometer 3 x 3 x 1 mm

The MMA8451, 2, 3Q has a selectable dynamic range of ±2g, ±4g, ±8g. The device has 8 different output data rates, selectable high pass filter cut-off frequencies, and high pass filtered data. The available resolution of the data and the embedded features is dependant on the specific device.

Note: The MMA8450Q has a different memory map and has a slightly different pin-out configuration.

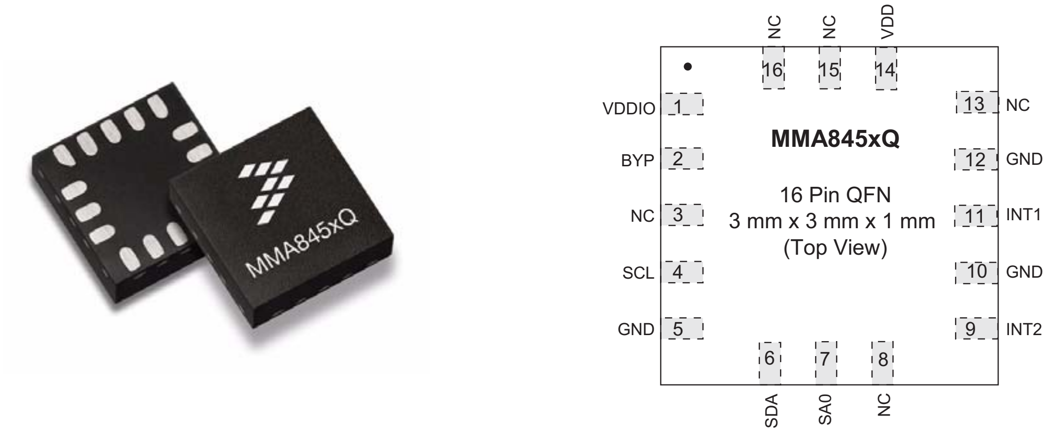

Figure 1. MMA8451, 2, 3Q Consumer 3-axis Accelerometer 3 x 3 x 1 mm

2.1 Output Data, Sample Rates and Dynamic Ranges of all Three Products

2.1.1 MMA8451Q

1. **14-bit data**
   2g (4096 counts/g = 0.25 mg/LSB) **4g** (2048 counts/g = 0.5 mg/LSB) **8g** (1024 counts/g = 1 mg/LSB)
2. **8-bit data**
   2g (64 counts/g = 15.6 mg/LSB) **4g** (32 counts/g = 31.25 mg/LSB) **8g** (16 counts/g = 62.5 mg/LSB)
3. **Embedded 32 sample FIFO (MMA8451Q)**

2.1.2 MMA8452Q

1. **12-bit data**
   2g (1024 counts/g = 1 mg/LSB) **4g** (512 counts/g = 2 mg/LSB) **8g** (256 counts/g = 3.9 mg/LSB)
2. **8-bit data**
   2g (64 counts/g = 15.6 mg/LSB) **4g** (32 counts/g = 31.25 mg/LSB) **8g** (16 counts/g = 62.5 mg/LSB)

2.1.3 MMA8453Q Note: No HPF Data

1. **10-bit data**
   2g (256 counts/g = 3.9 mg/LSB) **4g** (128 counts/g = 7.8 mg/LSB) **8g** (64 counts/g = 15.6 mg/LSB)
2. **8-bit data**
   2g (64 counts/g = 15.6 mg/LSB) **4g** (32 counts/g = 31.25 mg/LSB) **8g** (16 counts/g = 62.5 mg/LSB)

2.2 Application Notes for the MMA8451, 2, 3Q

The following is a list of all the application notes available for the MMA8451, 2, 3Q:

- **AN4068**, *Embedded Orientation Detection Using the MMA8451, 2, 3Q*
- **AN4069**, *Offset Calibration of the MMA8451, 2, 3Q*
- **AN4070**, *Motion and Freefall Detection Using the MMA8451, 2, 3Q*
- **AN4071**, *High Pass Data and Functions Using the MMA8451, 2,3Q*
- **AN4072**, *MMA8451, 2, 3Q Single/Double and Directional Tap Detection*
- **AN4073**, *Using the 32 Sample First In First Out (FIFO) in the MMA8451Q*
- **AN4074**, *Auto-Wake/Sleep Using the MMA8451, 2, 3Q*
- **AN4075,** ***How Many Bits are Enough? The Trade-off Between High Resolution and Low Power Using Oversampling Modes***
- **AN4076**, *Data Manipulation and Basic Settings of the MMA8451, 2, 3Q*
- **AN4077**, *MMA8451, 2, 3Q Design Checklist and Board Mounting Guidelines*

3.0 Number of Bits verses Effective Bits

The number of bits is normally specified as the number of bits of the digitizer. Any data acquisition system or ADC has inherent performance limitations. The data sheet may advertise that a part has a particular number of bits but the number of *effective* bits provides the insight into the resolution of the part. Every accelerometer will have a certain amount of system noise that will limit the number of effective bits. By understanding how to calculate and measure the noise performance of the part and by doing a few calculations the effective number of bits can be calculated to provide the resolution limitations of the device.

4.0 Noise

There are two types of noise in the accelerometer. There is electronic noise in the ASIC and there is mechanical noise from the MEMS g-cell. The electronic noise has been minimized as much as possible in the ASIC while the mechanical noise is due to thermo-mechanical noise of the moving parts. The noise in the MEMS g-cell is minimized by good design practices as well. The mechanical noise is a function of the resonant frequency, the mass m the damping Q and the temperature T as shown in the following equation:

$$N_{Mech} = \frac{\sqrt{\frac{4k_b T\omega}{mQ}}}{g}$$

The overall system noise is measured as a function of frequency. Often the Power Spectral Density (PSD) is given in the data sheet representing the total internal system noise. Spectral density captures the frequency content of a stochastic process (noise). This describes how the power is distributed with frequency. The noise in the accelerometer is predominantly considered Gaussian white noise and therefore is a constant value across all frequencies. The PSD given in the data sheet is given in units of μg per square root Hz. The PSD relates to the RMS noise by the following equations.

$$N^2_{RMS} = \int_0^{\infty} PSD(f)df$$

$$N^2_{RMS} = \int_0^{BW} PSD(f)df$$

$$N^2_{RMS} = PSD(BW - 0)$$

$$N_{RMS} = \sqrt{PSD \times BW}$$

From the above equations you can see that the PSD which is often given in the data sheet is actually the square root PSD. To calculate the RMS noise simply multiply the bandwidth by the PSD value. Note this is the RMS noise of the system but does not include the quantization noise from digitizing the signal.

4.1 Quantization Error

Quantization error occurs because the analog signal is sampled and must be divided into a finite number. The output of the ADC has 2^n divisions or counts, where n is the number of converter bits. A 12-bit ADC has 2^{12} or 4096 counts. The bit that represents the smallest change is the least significant bit (LSB). Each sample has a quantization error of ±0.5 of the LSB due to the difference between the true analog output and the count represented by the ADC output. The quantization error is the minimum noise level to resolve the signal in an ideal perfect system.

The “mean square” of the quantization error is found by integrating over the quantization voltage error range as follows:

$$QE_{mean^2} = \int_{-V_q/2}^{+V_q/2} \frac{E^2}{V_q} dE = \frac{V_q^2}{12}$$

The RMS value of the quantization error is found by taking the square root of the mean square error where *Vq* is the quantization voltage and is equal to 1 bit.

$$QE_{RMS} = \sqrt{\frac{V_q^2}{12}} = \frac{V_q}{\sqrt{12}} = \frac{1LSB}{\sqrt{12}} = 0.288 \cdot mg/LSB$$

The quantization noise is often so small that it is negligible in the calculation, but it is part of the overall noise that is measured.

4.2 Measuring Noise from the Accelerometer

The noise of the accelerometer can be *measured* by the following steps:

1. Place the accelerometer and ensure that it remains motionless away from any kind of vibration.
2. Datalog the raw data output from the XYZ axes of the accelerometer
3. Calculate the standard deviation from the data.
4. Calculate the RMS noise based on the known sensitivity.
5. Calculate the pk-pk noise to quantify the resolution

The **standard deviation** is from the following formula:

$$\sum_{i=0}^{n} \frac{1}{n}\sqrt{(x_i - \mu)^2} \qquad \text{where } \mu \text{ is the mean}$$

If the sensitivity of the part is 1024 counts/g then this equates to 0.976 mg/count. As an example if the standard deviation is 4.1 counts then from this value we can calculate the RMS Noise from the following equation:

N_{RMS} = STD* 1/Sensitivity

N_{RMS} = 4.1counts* 0.976 mg/count = 4.0016 mg

Note: The measured noise includes all noise of the system. This will include the analog noise and the quantization noise as well as any mechanical noise.

Knowing the N_{RMS} the PSD can be estimated if the bandwidth is known.

$$PSD = \frac{N_{RMS}}{\sqrt{BW}}$$

In this example if the bandwidth is 200 Hz (400 Hz sample rate) the PSD estimate would be $283\mu g/\sqrt{Hz}$.

If we multiply the N_{RMS} by 4 we get an estimate of the pk-pk noise which provides the resolution. Therefore the resolution estimate in this example would be 4.0016 mg*4, which is 16 mg.

To estimate the resolution from the data sheet, use the PSD value from the data sheet.

$$N_{RMS} = PSD \times \sqrt{BW}$$

The PSD value in the data sheet as mentioned is actually the square root PSD from the derivation above. This will give you the RMS noise of the system but does not include the quantization noise.

For example, if the accelerometer has a $PSD = 85\mu g/\sqrt{Hz}$ and the bandwidth is 200 Hz (Sample Rate = 400 Hz) then the RMS noise would be 1.2 mg and the pk-pk estimate would be 1.2*4 = 4.8 mg.

It follows that assuming the PSD is constant and the sensitivity is constant that the standard deviation measured should decrease as the bandwidth decreases and ultimately the RMS noise decreases.

5.0 Calculating the Effective Number of Bits

Data sheets on the website will advertise 16-bit resolution, but often times this is not true. The device may have 16 bits but the real question is how many bits are noise free? Calculating the effective number of bits the Signal to Noise Ratio (SNR) is equated. The RMS value of the signal is divided by the quantization noise.

$$\frac{S_{RMS}}{N_{RMS}} = \frac{\frac{2^n}{2\sqrt{2}}}{\frac{1}{\sqrt{12}}} = 2^n \cdot \frac{\sqrt{3}}{\sqrt{2}}$$

Then the signal to noise ratio is converted to the decibel system.

$$\left(\frac{S_{RMS}}{N_{RMS}}\right)db = 20\log\left(2^n \cdot \frac{\sqrt{3}}{\sqrt{2}}\right)$$

$$\left(\frac{S_{RMS}}{N_{RMS}}\right)db = 20\log 2^n + 20\log 1.225$$

$$\left(\frac{S_{RMS}}{N_{RMS}}\right)db = 602n + 1.76$$

$$n = \frac{SNR(db) - 1.76}{6.02}$$

This results in the equation for finding the effective number of bits “n”.

5.1 Example Finding Effective Number of Bits

If the accelerometer has a dynamic range of 2g, then the full signal is ±2g which is a total of 4g. The next equation relates how to calculate the SNR. The noise can be calculated based on measured data from the device from the steps provided in Section 4.2, or an estimate can be made if the PSD value and bandwidth is known.

$$SNR(db) = 20\log\left(\frac{\frac{4g}{2\sqrt{2}}}{N_{RMS}}\right)$$

$$n = \frac{SNR(db) - 1.76}{6.02}$$

If the result is that the device has 8.5 effective bits then by taking the full scale and dividing by 2^BEff, results in the true resolution (noise free).

- 4g/ 2^8.5 = 11.05 mg resolution

Now that there is knowledge on how to calculate resolution the question becomes more an understanding of the needs for the application.

This is something that the user needs to determine about their application. A lot of times this comes about from experimentation and creative use-case analysis. Freescale offers the Sensor Toolbox demo board system to allow customers to try any of our accelerometers to datalog and experiment with their application. It allows for data collection to analyze the performance and this application note is intended to be used to determine the true resolution of the devices.

6.0 Improving the Resolution using Oversampling and Decimation

The oversampling factor k is equal to 2^{2n}, where n is the number of bits to gain by oversampling. The signal to noise ratio increases as follows by adding on the 10logk term.

$$SNR[dB] = 6.02N + 1.76 + 10\log k$$

The following table shows the values for k with the SNR improvement and the added extra effective number of bits. This isn't an endless process. There is a trade-off with reaction time. As more and more samples are average the reaction time slows down by a factor proportional to "k".

Table 1. Oversampling Factor for Added Resolution

Oversampling Factor (k)	SNR Improvement (dB)	Extra Resolution Bits (n)
2	3	0.5
4	6	1.0
8	9	1.5
16	12	2.0
32	15	2.5
64	18	3.0

There is a trade-off between current consumption and resolution. The MMA8451, 2, 3 accelerometers have been designed to fit as many applications and markets as possible. The MMA845xQ series of accelerometers have been designed such that the same accelerometer can be used for high resolution applications or for low power applications simply by switching the device into a different mode. There is also an option for trading off the full dynamic range for improved noise performance.

The internal sampling rate is 1600 Hz. The oversampling ratio "OS ratio" represents the number of averaged samples. Note in High Resolution mode as many samples as possible are averaged. In Low Power mode only 2 samples are averaged. The device is power cycled at 400 Hz and below to minimize current consumption in Low Power mode. There are four different modes that the device can be in. These are Normal Mode, High Resolution Mode, Low Noise and Low Power Mode and Low Power Mode. The difference between the modes is seen in Tables 3, 4, 5 and 6 by the number of averaged samples at each ODR. The oversampling mode can be set for the Wake Mode as well as for the Sleep Mode. The Oversampling Mode register is found in CTRL_REG2. The MMA845xQ also has a low noise bit that can be set in CTRL_REG1. The low noise bit will improve the noise performance and this is separate from averaging samples. This bit will increase the sensitivity but the dynamic range will be limited from 8g to approximately 4g for all functions.

Table 2. MODS Oversampling Mode

(S)MODS1	(S)MODS0	Power Mode
0	0	Normal
0	1	Low Noise and Low Power
1	0	High Resolution
1	1	Low Power

The typical noise calculated in mg RMS is given for all different available sample rates when the Low Noise bit is set and also when it is cleared. This is done at all sample rates and at all oversampling modes. The equivalent effective number of noise free bits of the 14-bit data is given for each based on tested results. For an application requiring the highest resolution with the lowest current consumption, the low noise bit should be set and a trade-off will need to be made depending on acceptable requirements.

Note: The data in Tables 3, 4, 5 and 6 were taken in a typical applications lab environment without any sophisticated noise isolation during measurements.

Table 3. Normal Mode: MODS = 00 Oversampling Effective Bits and Noise Calculations All ODRs MMA8451Q

ODR	Noise RMS (Low Noise Bit = 1)	BEff (2g) (Low Noise Bit = 1)	BEff (4g) (Low Noise Bit = 1)	Noise RMS (Low Noise Bit = 0)	BEff (2g) (Low Noise Bit = 0)	BEff (4g) (Low Noise Bit = 0)	BEff (8g) (Low Noise Bit = 0)	OS Ratio	Current µA
1.56	0.233 mg	12.2	13.2	0.307 mg	11.8	12.8	13.8	128	24
6.25	0.466 mg	11.2	12.2	0.613 mg	10.8	11.8	12.8	32	24
12.5	0.659 mg	10.7	11.7	0.867 mg	10.3	11.3	12.3	16	24
50	1.32 mg	9.7	10.7	1.74 mg	9.3	10.3	11.3	4	24
100	1.32 mg	9.7	10.7	1.74 mg	9.3	10.3	11.3	4	44
200	1.32 mg	9.7	10.7	1.74 mg	9.3	10.3	11.3	4	85
400	1.32 mg	9.7	10.7	1.74 mg	9.3	10.3	11.3	4	165
800	1.86 mg	9.2	10.2	2.45 mg	8.8	9.8	10.8	2	165

Table 4. High Resolution Mode: MODS = 10 Oversampling Effective Bits and Noise Calculations All ODRs MMA8451Q

ODR	Noise RMS (Low Noise Bit = 1)	BEff (2g) (Low Noise Bit = 1)	BEff (4g) (Low Noise Bit = 1)	Noise RMS (Low Noise Bit = 0)	BEff (2g) (Low Noise Bit = 0)	BEff (4g) (Low Noise Bit = 0)	BEff (8g) (Low Noise Bit = 0)	OS Ratio	Current µA
1.56	0.0824 mg	13.7	14.7	0.108 mg	13.3	14.3	15.3	1024	165
6.25	0.164 mg	12.7	13.7	0.217 mg	12.3	13.3	14.3	256	165
12.5	0.233 mg	12.2	13.2	0.307 mg	11.8	12.8	13.8	128	165
50	0.466 mg	11.2	12.2	0.614 mg	10.8	11.8	12.8	32	165
100	0.659 mg	10.7	11.7	0.868 mg	10.3	11.3	12.3	16	165
200	0.932 mg	10.2	11.2	1.23 mg	9.8	10.8	11.8	8	165
400	1.32 mg	9.7	10.7	1.74 mg	9.3	10.3	11.3	4	165
800	1.86 mg	9.2	10.2	2.45 mg	8.8	9.8	10.8	2	165

Table 5. Low Noise and Low Power: MODS=01 Oversampling Effective Bits and Noise Calculations All ODRs MMA8451Q

ODR	Noise RMS (Low Noise Bit = 1)	BEff (2g) (Low Noise Bit = 1)	BEff (4g) (Low Noise Bit = 1)	Noise RMS (Low Noise Bit = 0)	BEff (2g) (Low Noise Bit = 0)	BEff (4g) (Low Noise Bit = 0)	BEff (8g) (Low Noise Bit = 0)	OS Ratio	Current µA
1.56	0.466 mg	11.2	12.2	0.614 mg	10.8	11.8	12.8	32	8
6.25	0.932 mg	10.2	11.2	1.23 mg	9.8	10.8	11.8	8	8
12.5	1.32 mg	9.7	10.7	1.74 mg	9.3	10.3	11.3	4	8
50	1.32 mg	9.7	10.7	1.74 mg	9.3	10.3	11.3	4	24
100	1.32 mg	9.7	10.7	1.74 mg	9.3	10.3	11.3	4	44
200	1.32 mg	9.7	10.7	1.74 mg	9.3	10.3	11.3	4	85
400	1.32 mg	9.7	10.7	1.74 mg	9.3	10.3	11.3	4	165
800	1.86 mg	9.2	10.2	2.45 mg	8.8	9.8	10.8	2	165

Table 6. Low Power: MODS = 11 Oversampling Effective Bits and Noise Calculations All ODRs MMA8451Q

ODR	Noise RMS (Low Noise Bit = 1)	BEff (2g) (Low Noise Bit = 1)	BEff (4g) (Low Noise Bit = 1)	Noise RMS (Low Noise Bit = 0)	BEff (2g) (Low Noise Bit = 0)	BEff (4g) (Low Noise Bit = 0)	BEff (8g) (Low Noise Bit = 0)	OS Ratio	Current µA
1.56	0.659 mg	10.7	11.7	0.868 mg	10.3	11.3	12.3	16	6
6.25	1.32 mg	9.7	10.7	1.74 mg	9.3	10.3	11.2	4	6
12.5	1.86 mg	9.2	10.2	2.45 mg	8.8	9.8	10.8	2	6
50	1.86 mg	9.2	10.2	2.45 mg	8.8	9.8	10.8	2	14
100	1.86 mg	9.2	10.2	2.45 mg	8.8	9.8	10.8	2	24
200	1.86 mg	9.2	10.2	2.45 mg	8.8	9.8	10.8	2	44
400	1.86 mg	9.2	10.2	2.45 mg	8.8	9.8	10.8	2	85
800	1.86 mg	9.2	10.2	2.45 mg	8.8	9.8	10.8	2	165

The trade-offs between low power and high resolution can be made by choosing the appropriate oversampling mode. These embedded options save the applications processor from averaging data to get improved performance and adds additional flexibility into the performance of the sensor. Example code for setting different oversampling modes is given in Freescale application note AN4074, Auto-Wake/Sleep Using the MMA8451, 2, 3Q.

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